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ON COMPLETELY PRIME AND COMPLETELY SEMI-PRIME IDEALS IN Γ-NEAR-RINGS

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ABSTRACT

In this paper we considered the algebraic system Γ-near-rings that was introduced by Satyanarayana. "Γ-near-ring" is a more generalized system than both near-ring and gamma ring. The aim of this short paper is to study and generalize some important results related to the concepts: completely prime and completely semi-prime ideals, in Γ-near-rings. We included examples when ever necessary.

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1. Introduction

In recent decades interest has arisen in algebraic systems with binary operations addition and multiplication satisfying all the ring axioms except possibly one of the distributive laws and commutativity of addition. Such systems are called "Near-rings". A natural example of a near-ring is given by the set M(G) of all mappings of an additive group G (not necessarily abelian) into itself with addition and multiplication defined by

$(f + g)(a) = f(a) + g(a)$; and

$(fg)(a) = f(g(a))$ for all $f, g \in M(G)$ and $a \in G$.

The concept Γ-ring, a generalization of 'ring' was introduced by Nobusawa [4] and generalized by Barnes [1]. Later, Satyanarayana [8, 9], Satyanarayana, Pradeep Kumar & Srinivasa Rao [14] also contributed to the theory of Γ-rings. A generalization of both the concepts near-ring and the Γ-ring, namely Γ-near-ring was introduced and studied by Satyanarayana [9, 11, 12], and later studied by several authors like: Booth [2], Booth & Groenewald [3], Syam Prasad [16].

Now, we collect some existing fundamental definitions and results which are to be used in later sections.

1.1 Definition: An algebraic system (N, +, .) is called a *near-ring* (or a right near-ring) if it satisfies the following three conditions:

(i) (N, +) is a group (not necessarily Abelian);

(ii) (N, .) is a semigroup; and

(iii) $(n_1 + n_2)n_3 = n_1n_3 + n_2n_3$ (right distributive law) for all $n_1, n_2, n_3 \in N$.

In general n.0 need not be equal to 0 for all n in N. If a near-ring N satisfies the property $n.0 = 0$ for all n in N, then we say that N is a zero-symmetric near-ring.

1.2. Definitions: A normal subgroup I of (N, +) is said to be

(i) a *left ideal* of N if $n(n^1 + i) - nn^1 \in I$ for all $i \in I$ and $n, n^1 \in N$

(Equivalently, $n(i + n^1) - nn^1 \in I$ for all $i \in I$ and $n, n^1 \in N$);

(ii) a *right ideal* of N if $IN \subseteq I$; and

(iii) an *ideal* if I is a left ideal and also a right ideal.

If I is an ideal of N then we denote it by $I \trianglelefteq N$.

1.3. Definitions: (i) An ideal (left ideal) P of N (with $P \neq N$) is said to be a *prime (prime left) ideal* of N if it satisfies the condition: I, J are ideals (left ideals) of N, $IJ \subseteq P$, implies $I \subseteq P$ or $J \subseteq P$.

(ii) An ideal P of N is said to be *completely prime* if for any $a, b \in N$, $ab \in P \Rightarrow a \in P$ or $b \in P$

(iii) An ideal S of N is said to be *semi-prime* if for any ideal I of N, $I^2 \subseteq S$ implies $I \subseteq S$.

(iv) An ideal S of N is said to be *completely semi-prime ideal* if for any element $a \in N$, $a^2 \in S$ implies either $a \in S$.

1.4. Definitions: (i) For any proper ideal I of N, the intersection of all prime(Completely Prime, respectively) ideals of N containing I, is called the *prime*(Completely Prime, respectively) *radical* of I and is denoted by P-rad(I) (C-rad(I) , respectively).

(ii) The Prime (Completely Prime, respectively) radical P-rad(0)(C-rad(0) , respectively) is also called as Prime (Completely Prime, respectively) radical of N and we denote this by P-rad(N) (C-rad(N), respectively).

For some other fundamental definitions and results, we refer Pilz [5], Satyanarayana [9, 13], Satyanarayana and Syam Prasad [15].

1.5. Definition: (Satyanarayna [9, 11, 12, 15]): Let (M, +) be a group (not necessarily Abelian) and Γ be a non-empty set. Then M is said to be a *Γ-near-ring* if there exists a mapping $M \times \Gamma \times M \to M$ (the image of (a, α, b) is denoted by $a\alpha b$), satisfying the following conditions:

(i) $(a + b)\alpha c = a\alpha c + b\alpha c$; and

(ii) $(a\alpha b)\beta c = a\alpha(b\beta c)$ for all $a, b, c \in M$ and $\alpha, \beta \in \Gamma$.

M is said to be a zero-symmetric Γ-near-ring if $a\alpha 0 = 0$ for all $a \in M$ and $\alpha \in \Gamma$, where 0 is the additive identity in M.

A natural example of Γ-near-ring is given below:

1.6 Example (Satyanarayana [11]): Let (G, +) be a non - abelian group and X be a non-empty set. Let $M = \{f / f: X \to G\}$. Then M is a group under point wise addition.

Since G is non-abelian, then (M, +) is non - abelian. Let Γ be the set of all mappings of G into X. If $f_1, f_2 \in M$ and $g \in \Gamma$, then, obviously, $f_1gf_2 \in M$. But $f_1g_1(f_2 + f_3)$ need not be equal to $f_1g_1f_2 + f_1g_1f_3$. To see this, fix $0 \neq z \in G$ and $u \in X$. Define G_u: $G \to X$ by $g_u(x) = u$ for all $x \in G$ and $f_z: X \to G$ by $f_z(x) = z$ for all $x \in X$. Now for any two elements $f_2, f_3 \in M$, consider $f_zg_u(f_2+ f_3)$ and $f_zg_uf_2 + f_zg_uf_3$. For all $x \in X$,
$[f_zg_u(f_2+ f_3)] (x) = f_z[g_u(f_2(x) + f_3(x))] = f_z(u) = z$ and
$[f_zg_uf_2 + f_zg_uf_3](x) = f_zg_uf_2(x) + f_zg_uf_3(x) = f_z(u) + f_z(u) = z + z$.
Since $z \neq 0$, we have $z \neq z + z$ and hence $f_zg_u(f_2+ f_3) \neq f_zg_uf_2 + f_zg_uf_3$.
Thus we have that M is a Γ-near-ring which is not a Γ- ring.

1.7. Definition: Let M be a Γ-near-ring. Then a normal subgroup I of (M, +) is called

(i) a *left ideal* if $a\alpha(b + i) - a\alpha b \in I$ for all $a, b \in M$, $\alpha \in \Gamma$ and $i \in I$;

(ii) a *right ideal* if $i\alpha a \in I$ for all $a \in M$, $\alpha \in \Gamma$, $i \in I$; and

(iii) an *ideal* if it is both a left and a right ideal.

Let M be a Γ-Near-ring and $\alpha \in \Gamma$. Satyanarayana [11] defined a binary operation "$*_\alpha$" on M by $a *_\alpha b = a\alpha b$ for all $a, b \in M$. Then $(M, +, *_\alpha)$ is a near-ring. So we may consider every element $\alpha \in \Gamma$ as a binary operation on M such that $(M, +, *_\alpha)$ is a near-ring. Also for any $\alpha, \beta \in \Gamma$, we have $(a *_\alpha b) *_\beta c = a *_\alpha (b *_\beta c)$ for all $a, b, c \in M$.

Conversely, if (M, +) is a group and Γ is a set of binary operations on M satisfying

(i) $(M, +, *)$ is a near-ring for all $* \in \Gamma$; and

(ii) $(a *_1 b) *_2 c = a *_1 (b *_2 c)$ for all $a, b, c \in M$ and for all $*_1, *_2 \in \Gamma$, then (M, +) is a Γ-near-ring.

1.8. Remark: (i) If $*_\alpha, *_\beta$ are operations on M with $a *_\alpha b = a *_\beta b$ for all $a, b \in M$, then the functions $*_\alpha, *_\beta$ are one and the same. So in this case, we have $*_\alpha = *_\beta$.

(ii) Suppose that (M, +) is a Γ-near-ring and also (M, +) is a Γ^*-near-ring with the following property: $\alpha \in \Gamma$ implies there exists $\beta \in \Gamma^*$ such that $a *_\alpha b = a *_\beta b$ for all $a, b \in M$. Then we may consider this case as $\alpha = \beta$ and so $\Gamma \subseteq \Gamma^*$.

1.9. Definition: Let (M, +) be a group. A Γ-near-ring M is said to be a *maximal Γ-near-ring* if M cannot be a Γ^*-near-ring for any $\Gamma \subset \Gamma^*$ (Here it is assumed that the restriction of the mapping $M \times \Gamma^* \times M \to M$ to $M \times \Gamma \times M$ is the mapping $M \times \Gamma \times M \to M$).

1. 10. Theorem (Th. 1.3 of Satyanarayana [11]): Let (M, +) be a group and P = {* / * is a binary operation on M such that (M, +, *) is a near-ring and M * M = M}. Then there exists a partition $\{\Gamma_i / i \in I\}$ of P such that (M, +) is a maximal Γ_i-near-ring for all $i \in I$. Conversely, if $\{\Gamma_j\}_{j\in J}$ be a disjoint collection of sets such that (M, +) is a maximal Γ_j-near-ring for each $j \in J$ with M * M = M for all $* \in \Gamma_j$ and for all $j \in J$, then $\bigcup_{j\in J} \Gamma_j \subseteq P$. Moreover (Say property B: If Γ is a nonempty set such that (M, +) is a maximal Γ-near-ring implies $\Gamma = \Gamma_j$ for some $j \in J$).

If property B holds, then $\bigcup_{j\in J} \Gamma_j = P$.

1.11. Definition: Let M be a Γ-near-ring and $\gamma \in \Gamma$. A subset A of M is said to be a *γ-ideal* of the Γ-near-ring M if A is an ideal of the near-ring $(M, +, *_\gamma)$.

1. 12. Observations: (i) Let (N, +, *) be a near-ring which is not zero symmetric. Then there exists $a \in N$ such that $a * 0 \neq 0$. Write $\Gamma = \{*\}$. Then N is a Γ-near-ring with $a\alpha 0 \neq 0$ for some $a \in N$, $\alpha \in \Gamma$. Therefore, in this case, N cannot be a zero symmetric Γ-near-ring.

(ii) Let M be a Γ-near-ring and (I, +) a normal subgroup of (M, +). It is clear that I is an ideal of the Γ-near-ring M if and only if I is an ideal of the near-ring $(M, +, *_\alpha)$ for all $\alpha \in \Gamma$. In other words, I is an ideal of the Γ-near-ring M if and only if I is a γ-ideal of M for all $\gamma \in \Gamma$.

(iii) Let M be a Γ-near-ring. For any $\Gamma^* \subseteq \Gamma$ we have that M is a Γ^*-near-ring. Every ideal I of the Γ-near-ring M is also an ideal of Γ^*-near-ring M, but the converse need not be true.

To see this, we observe the following example.

1. 13. Example: Consider G = {0, 1, …, 7} the group of integers modulo 8 and a set X = {a, b}. Write $M = \{f / f: X \to G$ such that $f(a) = 0\} = \{f_i / 0 \leq i \leq 7\}$ where $f_i: X \to G$ is defined by $f_i(b) = i$, $f_i(a) = 0$ for $0 \leq i \leq 7$. Consider two mappings g_0, g_1 from G to X defined by $g_0(i) = a$ for all $i \in G$, and $g_i(i) = a$ if $i \notin \{0, 3\}$, $g_1(3) = g_1(7) = b$.
Write $\Gamma = \{g_0, g_1\}$ and $\Gamma^* = \{g_0\}$. Now M is a Γ-near-ring and also Γ^*-near-ring.
Now $Y = \{f_0, f_2, f_4, f_6\}$ is an ideal of the Γ^*-near-ring M but not an ideal of the Γ-near-ring M (since $f_2 \in Y$ and $f_3g_1(f_1 + f_2) - f_3g_1f_1 = f_3 \notin Y$).

1.14. Definition: Let I be an ideal of N. Then a prime (completely prime, respectively) ideal of N containing I is called a *minimal prime* (*minimal completely prime, respectively*) *ideal* of I if P is minimal in the set of all prime (completely prime, respectively) ideals containing I.

1.15. Theorem (Th. 1.4 of [13]): Let I be an ideal of a near-ring N. Then I is a semi-prime ideal of a N $\Leftrightarrow$ I is the intersection of all minimal prime ideals of N $\Leftrightarrow$ I is the intersection of all prime ideals containing I.

1.16. Theorem (Cor. 5.1.10 of Satyanarayana [9]) : Let N be a near-ring and A an ideal of N. Then A is completely semi-prime ideal if and only if A is the intersection of completely prime ideals of N containing A.

1.17. Theorem (Theorem 2.2(b) of Satyanarayana [13]): An ideal P of N is prime and completely semi-prime $\Leftrightarrow$ it is completely prime.

1.18. Theorem (Lemma 2.7 of Satyanarayana [13]): Every minimal prime ideal P of a completely semi-prime ideal I is completely prime. Moreover, P is minimal completely prime ideal of I.

1.19.Theorem (Theorem 2.8 of Satyanarayana [13]): Let I be a completely semi-prime ideal of N. Then I is the intersection of all minimal completely prime ideals of I.

1.20. Theorem (Theorem 2.9 of Satyanarayana [13]): If P is a prime ideal and I is a completely semi-prime ideal, then P is minimal prime ideal of I if and only if P is minimal completely prime ideal of I.

1.21. Corollary: (Corollary 2.10 Satyanarayana [13]): If I is a completely semi-prime ideal of N, then I is the intersection of all completely prime ideals of N containing I.

2. γ-Completely Prime and γ-Completely Semi-prime γ-Ideals.

Throughout this section we consider only zero-symmetric right near-rings, and M denotes a Γ-near-ring.

2.1 Definition: Let $\gamma \in \Gamma$. A γ-ideal I of M is said to be

(i) *γ-completely prime* if $a, b \in M$, $a\gamma b \in I \Rightarrow a \in I$ or $b \in I$.

(ii) *γ-completely semi-prime* if $a \in M$, $a\gamma a \in I \Rightarrow a \in I$.

2.2 Note: Let M be a Γ-near-ring and $\gamma \in \Gamma$. Write N = M. Now $(N, +, *_\gamma)$ is a near-ring. Let I be a γ-ideal of M.

(i) I is a γ-completely prime γ-ideal of M if and only if I is a completely prime ideal of the near-ring $(N, +, *_\gamma)$.

(ii) I is a γ-completely semi-prime γ-ideal of M if and only if I is a completely semi-prime ideal of the near-ring $(N, +, *_\gamma)$.

2.3 Remark: Every γ-completely prime γ-ideal of M is a γ-completely semi-prime γ-ideal of M. [**Verification**: Let I be a γ-completely prime γ-ideal of M. Let $a \in M$. Suppose $a\gamma a \in I$. Since I is γ-completely prime, we have that $a \in I$. Thus I is a γ-completely semi prime γ-ideal of M.]

2.4 Corollary: Let M be a Γ-near-ring, $\gamma \in \Gamma$ and A be a γ-ideal of M. Then A is γ-completely semi-prime γ-ideal if and only if A is the intersection of γ-completely prime γ-ideals of M containing A.

Proof: A is γ-completely semi-prime γ-ideal $\Leftrightarrow$ A is completely semi-prime ideal of the near-ring $(M, +, *_\gamma)$ (by Remark 2.3) $\Leftrightarrow$ A is the intersection of all completely prime ideals of the near-ring $(M, +, *_\gamma)$ containing A (by Theorem 1.16) $\Leftrightarrow$ A is the intersection of all γ-completely prime γ-ideals of M containing A. The proof is complete.

2.5 Definition: Let A be a proper ideal of M. The intersection of all γ-completely prime γ-ideals of M containing A of M, is called as the *γ-completely prime radical of A* and it is denoted by C-γ-rad(A). The *γ-completely prime radical of M* is defined as the γ-completely prime radical of the zero ideal, and it is denoted by C-γ-rad(M).

2.6 Note: From Theorem 1.16, and Theorem 2.4 we conclude the following:
(i) An ideal A of a near-ring is completely semi-prime $\Leftrightarrow$ A = C-rad(A).

(ii) A γ-ideal A of a Γ-near-ring M is γ-completely semi-prime $\Leftrightarrow$ A = C-γ-rad(A).

2.7 Definitions: (i). A γ-ideal P of a Γ-near-ring M is said to be a *γ-prime γ-ideal* of M (with respect to $\gamma \in \Gamma$) if $A\gamma B \subseteq P$ for any two γ-ideals A, B of M implies $A \subseteq P$ or $B \subseteq P$.

(ii). A γ-ideal S of a Γ-near-ring M is said to be a *γ-semi-prime γ-ideal* of M (with respect to $\gamma \in \Gamma$) if $A\gamma A \subseteq S$ for any γ-ideal A of M implies $A \subseteq S$.

2.8 Note: Let P be an γ-ideal of a Γ-near-ring M and $\gamma \in \Gamma$. Then we have the following:

(i). P is a γ-prime γ-ideal of the Γ-near-ring M $\Leftrightarrow$ P is a prime ideal of the near-ring $(M, +, *_\gamma)$.

(ii). P is a γ-semi-prime γ-ideal of the Γ-near-ring M $\Leftrightarrow$ P is semi-prime ideal of the near-ring $(M, +, *_\gamma)$.

(iii).Suppose that S is a γ-ideal of M. Then (by Theorem1.15) we have that S is γ-semi-prime γ-ideal of M $\Leftrightarrow$ S is the intersection of all γ-prime ideals P of M containing S.

The following corollary follows from Theorem 1.17.

2.9 Corollary: A γ-ideal P of a Γ-near-ring M is γ-prime and γ-completely semi-prime $\Leftrightarrow$ it is γ-completely prime.

2.10 Definitions: Let I be a γ-ideal of a Γ-near-ring M for $\gamma \in \Gamma$.
I is called a *minimal γ-prime (γ-Completely Prime, respectively) γ-ideal* of M if it is minimal in the set of all γ-prime (*γ-Completely Prime, respectively*) γ-ideals containing I.

The following corollary follows from Theorem 1.18.

2.11 Corollary: Let P be a γ-ideal of a Γ-near-ring M for $\gamma \in \Gamma$. Every minimal γ-prime γ-ideal P of a γ-completely semi-prime γ-ideal I is a γ-completely prime γ-ideal. More over P is a minimal γ-completely prime γ-ideal of I. The following corollary follows from Theorem 1.19.

2.12 Corollary: Let $\gamma \in \Gamma$. If I is γ-completely semi-prime γ-ideal of M, then I is the intersection of all minimal γ-completely prime γ-ideals of I.

2.13 Corollary: Let $\gamma \in \Gamma$ and P be a γ-ideal of M. If P is a γ-prime γ-ideal and I is a γ-completely semi-prime γ-ideal, then P is a minimal γ-prime γ-ideal of I if and only if P is a minimal γ-completely prime γ-ideal of I.

Let $\gamma \in \Gamma$. By applying the Corollary 1.21 to the near-ring $(M, +, *_\gamma)$ we get the following.

2. 14 Corollary: Let $\gamma \in \Gamma$. If I is a γ-completely semi-prime γ-ideal of M, then I is the intersection of all γ-completely prime γ-ideals of M containing I (that is, I = $\cap$ {P / P is a γ-completely prime γ-ideal of M such that I $\subseteq$ M} = C-γ-rad(I)).

2.15 Example: Let us consider the Example 2.11 of Satyanarayana [13]. In this example, (G, +) is the Klein four group where G = {0, a, b, c}. We define multiplication on G as follows:

.	0	a	b	C
0	0	0	0	0
a	a	a	a	A
b	0	a	b	C
c	a	0	c	B

This (G, +, .) is a near-ring which is not zero symmetric. The ideal {0, a} is only the nontrivial ideal and also it is completely prime.

(i) Write M = G, the Klein four group and G = {0, a, b, c}. Define multiplication on G as above. If we write Γ = {**.**}, then M is a Γ-near-ring, which is not a zero symmetric Γ-near-ring (because $a\gamma 0 = a.0 \neq 0$). It is clear that for $\gamma \in \Gamma$, the γ-ideal {0, a} of M is only the nontrivial γ-completely prime γ-ideal. The γ-ideal (0) of M is γ-completely semi-prime γ-ideal, but not γ-completely prime γ-ideal (because $c\gamma a = c.a = 0$ and $a \neq 0 \neq c$). Hence the γ-completely semi-prime γ-ideal (0) can not be written as the intersection of its minimal γ-completely prime γ-ideals.

From this example 2.15, we can conclude that if M is not a zero symmetric Γ-near-ring, then the corollary 2.14 need not be true.

2.16 Notation: Let A be a γ-ideal of M. The intersection of all γ-prime ideals containing A is called the *γ-prime radical* of A and it is denoted by P-γ-rad(A). The γ-prime radical of M is defined as the γ-prime radical of the zero ideal (0). So P-γ-rad(M) = P-γ-rad(0).

2.17 Theorem: Let A be an ideal of M. Then

(i). P-γ-rad(A) is a γ-semi-prime γ-ideal.
(ii). The γ-prime radical of M is a γ-semi-prime γ-ideal.

Proof: Write S = P-γ-rad(A).

(i). Since S = P-γ-rad(A) is equal to the intersection of all γ-prime γ-ideals of M containing S, by Note 2.8(iii), it follows that S is a γ-semi-prime γ-ideal. Thus we conclude that the γ-prime radical of a γ-ideal A (that is, P-γ-rad(A)) is a γ-semi-prime γ-ideal.

(ii). Follows from (i), by taking A = (0).

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